Targeted Persuasive Advertising

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I. Introduction

“An important aspect of marketing practice is the targeting of consumer segments for different promotional activity.” (Rossi et al. 1996)

The door-to-door salesman has returned in electronic form. Thanks to the internet and improvements in data collection, modern marketers are once more able to tailor a sales pitch to an individual customer. A web-based retailer may offer different storefronts and products to customers with different tastes, and a company can send its clients e-mails with enticements to purchase in keeping with their recorded preferences. At present, this 'direct marketing' technology is in its adolescence, and spam (a common term for unsolicited commercial e-mail) is still largely a hit or miss affair - men receive enticements to breast enlargement and teenagers are asked if they wish a low interest rate on their mortgage. There is, however, every reason to believe that the accuracy of these methods will increase, and the current extent of their use bears witness to firms' faith in their efficacy. (Rossi et al. 1996) In 1999, direct marketing accounted for well over half of all marketing expenditures in the United States (Economist 1999).

The mechanics of data gathering via the internet is already impressive. Cookies, small bits of code transmitted by web sites, already allow advertisers to track consumers' browsing patterns, and legal, ubiquitous spyware uses methods similar to those of computer viruses to gather detailed information on the content and use of a computer on which they are installed, then transmit it to its home firm. The number of such programs has become a nuisance in its own right, and Spybot¹, a program designed to keep track of and eliminate spyware, had over 11,000 programs in its search database as of January 2004. These electronic researchers, the data collection and analysis programs, have numerous advantages over their flesh and blood kin. They do not sleep, take days off or earn wages. And most importantly to many firms, they do precisely what they're told.

¹ Hosted at http://www.safer-networking.org/
This begs the question - what is it, exactly, that firms will do with these possibilities, and this information? The companies themselves are still uncertain of how best to exploit the possibilities. They have gathered vast amounts of data on consumer habits and preferences, but are still at a loss as to how to use it. There is some worry, particularly noticeable among privacy advocates (Economist 2003) and European governments (Economist 2000) that powerful corporations will abuse their ability to influence consumers in such a way that diversity will be compromised in favour of homogeneity of preferences. Multinationals selling the same good to different cultures, they reason, will wish to do what they can to shape consumer tastes into a homogeneous mass favourable to the sale of their product.

The key issue causing the worry is that firms do not need to use their wealth of information to find a suitable market for their products, given preferences and the characteristics of the good. Instead, like the salesmen of yore, the electronic marketers can change their approach to suit the consumer, convincing them to buy something they would otherwise have done without. The French government is not worried that McDonald's will take the business of all its hamburger-loving clientele. It is worried that the fast food chain will use its wiles to charm the French away from *pate de fois gras* and to the Big Mac, convincing them in the process that they prefer fountain drinks to wine. When a firm knows with some accuracy what stimuli a consumer responds to, it can use these to make the customer fit the good - but only if it can target its messages: while Ginger may buy the soda endorsed by a prominent rap artist, it is not entirely clear that Aunt Maude will be likewise enticed.

Despite clear concern about the problem, there have been few detailed analyses (in the economic literature, at least) of the strategies involved in direct marketing. Indeed, there has been relatively little work done on persuasive advertising of any sort, as opposed to informative advertising. This neglect is partly the legacy of George Stigler and Gary Becker’s influential and eloquent assertion that “tastes, at least when held by an adult, are not capable of being changed by persuasion.” (Stigler and Becker 1977).
Economic discourse on advertising has instead focused on its role as a means for disseminating information.

Step right up!
Get your new screensaver!

The imagery from our advertising campaigns is so popular we've designed a screensaver calendar so you can continue to enjoy them right from your desktop.

>download

A cell phone ad campaign: http://www.telusmobility.com

Models of informative advertising assume that a consumer enters the world not knowing about the products within it, and must be informed of their characteristics and price through the issuing of advertisements. This is their sole purpose. While information is undoubtedly an important part of marketing (brand recognition, 'a name you can trust'), a cursory glance at actual advertisements will show that much of what is seen in them is not directly related to the features of the product in question. Abercrombie and Fitch, a clothing store, has run a marketing campaign starring notably unclothed and attractive ladies and gentlemen. A BC cell phone vendor's ads feature pigs, and little else.

The idea behind persuasive advertising is that a firm can somehow pay for advertisements that raise the valuation of a product in the eyes of its consumers. (A typology of the ways in which advertising may affect a consumer of differentiated products may be found in (von der Fehr and Stevik 1998).) An Abercrombie and Fitch shopper is enticed by the store's ad not because it reveals much about their clothing, but because the positive emotions experienced when viewing the ads are transferred to the company that created them, and the products that they sell. Most
models of persuasive advertising assume that ads are sent to the market as a whole, or to a sizeable portion of that market.

Economists have scarcely looked at targeted advertising, that is, a framework where the ads sent depend upon the characteristics of individual consumers. The most thorough paper on the phenomenon to date (Esteban et al. 2001) focuses on the case of a monopolist engaged in informative advertising. The firm may choose between advertising cheaply to a large number of consumers with mixed tastes, or paying more to advertise in a medium with a smaller audience whose tastes are more favourable to its product. In equilibrium, the monopolist chooses a higher level of advertising than is socially optimal.

As might be expected, marketers are more prolific in their writing about targeting. The technical discussion has focused largely on price discrimination and coupon issue. This is presumably because the psychology-inspired marketing language commonly used to write about persuasive advertising and its effects is difficult to reconcile with the more mathematical approach taken by this subject. Examples of the genre are a recent research note (Chen and Iyer, 2002) and an article on the profitable use of purchase history data (Rossi et al. 1996). The former paper uses a framework similar to the present one to address customized pricing under costly information acquisition.

The absence of an analysis of the strategic uses of persuasive advertising is an important gap in our knowledge, since as D.P.T. Young (Young 2000) has argued, the existence of persuasive advertising complicates the matter of defining a market, and must be taken into account when formulating regulatory policy. If persuasive advertising is targeted, then this provides firm with a flexible tool with which to manipulate preferences and through them, profits. Young suggests that the market power of a firm may be

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2 Given the possibility of perfect price discrimination through targeting, why should a firm bother with persuasive advertising? One reason is the possibility of arbitrage. If consumers may communicate and trade with each other (possibly through a centralized system such as e-bay), then these coupons and discounts could be arbitraged to the issuing firm’s detriment. Persuasive advertising does not have this problem, and this may explain much of its appeal even to those firms best placed to use ‘customized pricing’. The relative merits of each technique of consumer targeting will vary with the type of product under consideration.
usefully measured not only by the ability to maintain a significant price increase, but also
by its capability to gain an advantage over its competitors by creating an asymmetry in
demand.

In the following pages I endeavor to show that, under plausible conditions, direct
marketing by one of two duopolists can be worse for consumers than a monopoly.
Targeted ads can be used as a precise tool to facilitate collusion; a very small and
judicious marketing campaign may leave both firms better off than in the absence of
advertising. Even when the non-advertising firm is worse off, as would be expected, it
will do as well as a monopolist with its given market share. Moreover, under general
conditions the fear of homogenization of preferences is well-founded in the case of
monopoly.

My approach is most closely related to the economic literature on persuasive
advertising and product differentiation surveyed in (von der Fehr and Stevik 1998) and
the marketing literature on customized pricing exemplified by (Chen and Iyer 2002). As
in both traditions, I use the Hotelling (Hotelling 1929) framework as a basis for analysis.
I differ from the former in using targeting, the latter in using advertising, and both on
examining the possibility of cultural homogenization.

Section two restates the Hotelling model and introduces terminology. Section
three examines the advertising choice of a monopolist with direct marketing technology.
Section four analyses the case of duopoly, with only one firm advertising, and section
five concludes and lists directions for further work.
II. Benchmarks: The Hotelling model

The framework for the analysis will be a version of Hotelling’s 1929 model of differentiated products. Two firms are located at opposite ends of a line of length one. Firm 1 is at address zero, and Firm 2 is at address one. A unit mass of consumers is uniformly distributed along this line, and they are arranged according to their preferences for the goods. The utility for the consumer at address \( r \) from each firm’s product is given by the expressions

\[
U_r^1(p_1, r) = V - p_1 - tr, \text{ for firm 1’s product, and}
\]

\[
U_r^2(p_2, r) = V - p_2 - t(1 - r) \text{ for firm 2’s product}
\]

Here, \( V \) is an intrinsic valuation, \( t \) is a positive taste parameter (Hotelling’s ‘transport cost’), and \( p_i \) is the price charged by firm \( i \). Each consumer buys one of good 1, good 2, or an outside good providing zero utility.

Producers have no marginal or fixed costs. Profits are given by

\[
\pi_1 = p_1 x
\]

\[
\pi_2 = p_2 (1 - x)
\]

where \( x \) is the address at which a consumer is indifferent between good one and good two, and weakly prefers both to the outside good. Consumers to the right of \( x \) will buy from Firm 2, and consumers to the left of \( x \) will buy from Firm 1. This boundary point is defined by

\[
V - p_1 - tx = V - p_2 - t(1 - r)
\]

\[
x = \frac{1}{2} \left( 1 + \frac{p_2 - p_1}{t} \right)
\]
and is subject to the standard constraint that the utility of goods 1 and 2 at \( x \) be weakly greater than that of the outside good, so that the market is covered.

Solving the first order conditions, we find

\[
p_1 = p_2 = t \\
x = \frac{1}{2} \\
\pi_1 = \pi_2 = \frac{t}{2}
\]

And the constraint becomes \( V - \frac{3}{2}t \geq 0 \), or \( \frac{V}{t} \geq \frac{3}{2} \). This means that transport costs must be sufficiently small with respect to the intrinsic valuation of the good for all consumers to buy one of the two varieties offered.

Producers’ surplus is \( t \), and consumers’ surplus is given geometrically by

\[
CS = \left( V - \frac{3}{2}t \right) + 2 \frac{1}{2} \left[ (V-t) - \left( V - \frac{3}{2}t \right) \right] = V - t
\]

The case of monopoly is solved similarly, save that the rival supplies the outside good. Assuming firm 1 is the monopolist, the monopoly’s market share \( x^M \) is given by
\[ V - p_1^M - t x^M = 0 \]

\[ x^M = \frac{V - p_1^M}{t} \]

The superscript M denotes monopoly values. The firm’s profits are given by

\[ \pi^M = p_1^M x^M \]

Solving the first-order conditions, we find

\[ p_1^M = \frac{V}{2} \]

\[ x^M = \frac{V}{2t} \]

\[ \pi^M = \frac{V^2}{4t} \]

In this case, the market need not be covered. It will be covered if and only if \( \frac{V}{t} \geq 2 \). For \( \frac{V}{t} > 2 \), the monopolist has the entire line as its market share, and both its price and profits are equal to \( V-t \), the indifference price for the consumer at address 1.
Given \( \frac{V}{t} \leq 2 \), that is, that \( x^M \leq 1 \), consumers’ surplus is given geometrically by

\[
CS^M_B = \frac{1}{2} \left( V - \frac{V}{2} \right) \frac{V}{2t} = \frac{V^2}{8t}
\]

Consumers’ surplus is equal to half of producer’s surplus. The ‘B’ denotes a benchmark value.

Total surplus is

\[
TS^M_B = \frac{3 V^2}{8t}
\]

For \( \frac{V}{t} > 2 \), consumers’ surplus is found by similar methods to be \( \frac{t}{2} \).

The above models will be the benchmarks against which the effects of advertising will be compared.

### III. Targeted Persuasive Advertising in a Monopoly

**Definition:** *Targeted Persuasive Advertising is a technology that allows a firm to pay \( cM_r, c>1 \), to raise the valuation of its good to the consumer at address \( r \) by \( M_r \).*

As an example, suppose a given consumer is willing to pay up to 6 for a good whose price is 10. Without advertising, the consumer would decline to buy this good. Now suppose that the producer has access to advertising as above, and the cost parameter, \( c \), is equal to 2. By paying 8, the firm is able to raise the consumer’s reservation price by 4.
The consumer now values the good at 10, and will buy it, resulting in a profit of 2 for the company.

Let us look at TaPA in the context of a Hotelling monopoly. Consumer utility is now given by

\[ U^A_M(r) = V - p^A_M - tr + M_r(r) \]

where \( p^A_M \) denotes the price charged by the firm. The monopolist’s profits are

\[ \pi^A_M = p^A_M x^A_M - \int_0^1 M_r(r)dr \]

The monopolist’s task is to set the price choose an advertising scheme.

**Proposition 1:** For a given price, optimal advertising involves raising all negative-valued preferences to zero until an address \( b \leq 1 \). That is,

\[ M^*_r(r) = \begin{cases} A(r) & a < r < b \\ 0 & \text{otherwise} \end{cases} \]

\[ A(r) \equiv -(V - p^A_M - tr) \]

The points \( a \) and \( b \) are defined by

\[ V - p^A_M - ta = 0 \]

\[ cM^*_r(b) = p^A_M \]
The intuition behind this is as follows. Given a price such that the monopolist does not cover the market, it will always be profitable to advertise to the customers just to the right of the indifference point. As in the example above, by topping up preferences to the point of indifference, the monopolist can gain the whole of the price as revenue for the cost of the advertising. At some point, called \( b \), the cost of advertising will be weakly greater than its benefit. The firm will not advertise beyond this point. It will also forego advertising to those consumers whose valuations are higher than the price it charges, since this would be a wasteful expenditure.

**Lemma 1:** If \( M^*_r(r) > 0 \), then \( U^*_M(r) = 0 \) (that is, \( M^*_r(r) = A(r) \)).

**Proof:** Proof by contradiction.

Suppose

\[
U^*_M(r) > 0 \quad \text{and} \quad M^*_r(r) > 0
\]

for some \( r \). The consumer at \( r \) will buy the good. Further, there exists some \( \varepsilon > 0 \) such that \( U^*_M(r) - \varepsilon > 0 \), and so \( M^*_r(r) \) could fall to \( M^*_r(r) - \varepsilon \) and the consumer at \( r \) would still buy the good at the given price. Since this would result in an increase in profits for the monopolist, the stated case cannot be optimal.

Now suppose that

\[
U^*_M(r) < 0 \quad \text{and} \quad M^*_r(r) > 0.
\]

In this case, the consumer will not buy the good. Since the same result could be obtained at zero cost by not advertising, this cannot be optimal. Hence, if \( M^*_r(r) > 0 \), it must be the case that \( U^*_M(r) = 0 \) identically, \( q.e.d. \).
**Corollary 1**: If there exists an $0 \leq a \leq 1$ such that $U^*_M(a) = 0$ when $M_r(a) = 0$, then $M^*_r(r) = 0$ for all $r \leq a$.

This follows from the above and the fact that the level of advertising can never be negative. The corollary establishes the left end point for the support of the advertising campaign.

What of the right end point?

**Lemma 2**: If $cA(r) > p^*_M$, then $M^*_r(r) = 0$.

**Proof**: Suppose $cA(y) > p^*_M$ for some $0 \leq y \leq 1$, and $M^*_r(r) > 0$. Then reducing $M^*_r(r)$ to zero would increase firm profits, and so it cannot be optimal. Since advertising cannot be negative, the lemma holds.

**Lemma 3**: If $cA(r) < p^*_M$ and $r > a$, then $M^*_r(r) > 0$

Suppose not. That is, suppose that $r > a$ and $M^*_r(r) = 0$. Since $U^*_M(r)$ falls with $r$, by the definition of $a$, we must have $U^*_M(r) < 0$. That is, the consumer at $r$ will not buy the monopolist’s product. The monopolist could increase its profits by $(p^*_M - cA(r)) > 0$ by paying for the amount of advertising dictated in Lemma 1. Hence, zero advertising at this point cannot be optimal.

**Corollary 2**: The point $b$ defined by $cM^*_r(b) = p^*_M$ is the right end point of the support of the advertising campaign.

From Lemma 1, we see that $M^*_r(r)$ rises with $r$, and so if there exists a $0 \leq b \leq 1$ such that $cM^*_r(b) = p^*_M$, then using Lemma 2, $M^*_r(r) = 0$ for all $r > b$. 
Proof of Proposition 1: From Lemma 1, we see that within the support of advertising, it is optimally $A(r)$. From Corollary 1 and Lemma 3, we see that the left end point of the support is the point $a$, and Corollary 2 gives us the right end-point, $b$.

Given this advertising scheme, what does it look like?

The monopolist’s profits are given by

$$
\pi^A_M = p^A_M b - c \int_a^b M^*_r(r)dr
$$

Substituting for $a$, $b$ and $M^*_r(r)$ turns this into a function of $p^A_M$:

$$
\pi^A_M (p^A_M) = \frac{p^A_M}{t} \left( V + \left( \frac{1 - 2c}{2c} \right) p^A_M \right)
$$

Solving the first-order conditions, we find
These are reasonable results. The price is positive and greater than the no ad monopoly price for $c>1$. The support is such that $a<b$, and profits are greater than monopoly profits in the benchmark case. In order to have $b<1$, we require

$$\frac{V}{t} < 2 - \frac{1}{c}$$

The value of the right hand side ranges between 1 and 2, and fits well with previous restrictions on the ratio between intrinsic valuation and transport costs.

Note that $a \leq x_M \leq b$. That is, the support of the advertising campaign brackets the benchmark monopoly market share, with more of the market being covered than in the absence of ads. That $a$ is lower than $x_M$ follows from the price being higher when ads are present. A higher price will, in the absence of ads, drive the indifferent consumer towards the origin. That more of the market is covered follows from advertising making it possible to raise prices without lowering them for all consumers, in a manner similar to price discrimination\(^3\).

Geometrically, we find

\(^3\) Similar, but not congruent. Since advertising is particular to a consumer, it can be used in situations where arbitrage among customers would make price discrimination impossible. This may be the case in a computer-literate society where customers have access to an auction network where they may trade amongst themselves.
For $c>1$, this will always be less than the surplus in the case of no ads. Total surplus is

$$TS_A^M = \frac{1}{2} \frac{V^2}{t} \frac{(3c^2 - 3c + 1)}{(2c-1)^2}$$

Surprisingly, even given that advertising is costly, total surplus rises from the benchmark monopoly case:

$$TS_A^M - TS_B^M = \frac{1}{8} \frac{1}{t} \frac{V^2}{(2c-1)^2} > 0$$

The increase in price and market share dominate the effects of a costly transfer of surplus. The reason for this is, once more, the same effect illustrated by the numerical example used to introduce the advertising technology. While $c>1$ and thus advertising is costly, the firm only needs to ‘top up’ a consumer’s preferences, which by construction is cost-effective on the entirety of the support.

What about the claims of homogenization? From the optimal advertising scheme, it is clear that there will be some level of homogenization. How prevalent is it?

The fraction of the monopoly’s market that is advertised to, and who have thus been homogenized so as to obtain zero surplus, is inversely related to the cost’s of advertising:

$$\frac{b-a}{b} = \frac{1}{c}$$
Unless persuasive advertising is exceedingly costly, the advertising monopolist will indeed homogenize the preferences of a sizeable portion of its clientele.

**Section IV: Barriers**

When the advertising firm needs to take into account a rival as well as the outside good, the situation becomes more complicated.

At first blush, it might seem that the case of duopoly, with only one firm advertising, is a straightforward extension of the case of an advertising monopoly. Instead of providing zero surplus at the margin, the advertiser must provide a positive surplus equal to that granted by its rival at that address.

Points a and b are chosen as in the previous section. In what is now a two-period game, advertising is set in the first period and prices are simultaneously determined in the second.

Unfortunately, this procedure will not, in general, provide an equilibrium. Faced with a situation such as that shown above, the non-advertising firm (Firm 2) will have an incentive to deviate. This deviation may take two forms, as illustrated below:
Faced with M and starting from p2, Firm 2 may either lower its price to (say) p2' so as to bypass Firm 1’s advertising and gain market share, or raise its price to p2'' while keeping to its new market share. In the former case, its profit is the rectangle ADFE, and in the latter, CDKJ. One or both of these will, in general, provide greater profit than that from p2, that is, BDHG. Note that in the case of the price falling to p2’, or indeed to any level below p2, Firm 1’s advertising, M, is entirely wasted. The consumers in the support of the advertising (and beyond) will prefer Firm 2’s good to Firm 1’s. It is not enough, then, for Firm 1 to react to a given p2, as a monopolist reacts to a given outside good. It must take these strategic effects into account if its persuasion is to have a positive impact on profits.

Given the incentive for Firm 2 to deviate, the advertising firm will wish to engage in advertising for two different reasons: first, to prevent its rival from price-cutting its way past the advertising campaign, and secondly, to exploit the territory so secured through the methods of the previous section. That is to say, the first aim of an advertising scheme should be to persuade Firm 2 to restrict itself to the interval (b,1]. With this accomplished, Firm 1 may act as a monopolist on [0,b]. The former motive is the barrier motive for advertising; the second, the topup (in that preferences of consumers are topped up to some minimum level if they fall below it).
In the analysis that follows, I will concern myself only with the construction of a barrier and ignore the topoff motive. This is done because the intuition for topoff is exceedingly similar to that of the monopoly case, so that there is little to be gained by introducing it here. Moreover, I will show that even in the absence of topoff advertising, Firm 1 can do far better and consumers far worse than in the case of no advertising. Adding the possibility of homogenizing advertising on the blocked-off interval would only strengthen these results.

IV.1 Mirror Prices

How is this barrier created? Consider the case of no advertising. For any given $p_1$ such that all consumers weakly prefer good 1 to the outside good, the profits of Firm 2 are quadratic in $p_2$, or equivalently (and more usefully) in the market share of Firm 1, $x(p_1,p_2)$.

$$\pi_2(p_1, p_2) = p_2(1 - x(p_1, p_2))$$

The indifference point is defined by the condition

$$V - t(x) - p_1 = V - t(1 - x) - p_2$$

from which we see that

$$p_2 = p_1 + t(2x - 1)$$

Given Firm 1’s price, Firm 2’s price is a linear function of $x$. Using the above and substituting for $p_2$ as a function of $x$, we have

$$\pi_2(x(p_1, p_2), p_1) = (p_1 - t) + (3t - p_1)x(p_1, p_2) - 2tx(p_1, p_2)^2$$
The equivalent expression in prices is

\[
\pi_2(p_2) \mid p_1 = \frac{1}{2} \left( 1 + \frac{p_1}{t} \right) p_2 - \frac{1}{2t} p_2^2
\]

For a fixed \( p_1 \), these are quadratic in \( x \) and \( p_2 \), respectively, as shown below:

\[
p_2^* = \frac{1}{2} (t + p_1)
\]

The diagram in term of \( x \) is similarly shaped.

This implies that, given \( \pi_2 \geq 0 \),

\[
\forall p_2 > p_2^* \exists p_2' (p_2) < p_2 \text{ s.t. } \pi_2 \left( p_2' (p_2) \right) \mid p_1 = \pi_2 (p_2) \mid p_1
\]

For any price greater than Firm 2’s optimal price, there exists a lower, ‘mirror’ price at which the firm can earn an equal profit by trading price for market share\(^4\). The lower mirror price implies a lower \( x \) and hence a lower market share for Firm 1.

\(^4\) At this lower price, and given \( p_1 \), Firm 2’s market share may well be constrained to be unity. This does not change the existence of the mirror price, though it will somewhat alter its calculation.
The ‘mirror price’ result continues to hold in the case where \( p_1 \) does not cover the consumer line, but only within certain restrictions. Suppose that \( p_1 \) is such that (only) consumers on \((b,1]\), \(0<b<1\), prefer the outside good to good 1. Then

\[
V - tb - p_1 = 0
\]

\[
p_1 = V - tb
\]

For \( x \leq b \), the diagram is the same as above. For \( x>b \), however, it no longer applies. Since consumers to the right of \( b \) prefer the outside good, Firm 2 can act as a monopolist on that interval\(^5\).

The graph has a kink at \( b \), as shown below. Before \( b \), the duopoly profit function applies, and after \( b \), the monopoly profit function is relevant.

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\(^5\) An example might be helpful. Suppose that Firm 1’s price is such that the consumer at address 1/3 is indifferent between good 1 and the outside good. That means that all consumers to the right of 1/3 will prefer the outside good to good 1. Now think of Firm 2, located at address 1. The firm must decide, given Firm 1’s price, which market share it desires (or equivalently, what price it wishes to charge). If it chooses a market share between 2/3 and 1, say, 7/8, then it must provide the consumer at address 1/8 with the same, positive surplus she would obtain from consuming Firm 1’s good. Since Firm 2 cannot advertise or price-discriminate, this determines the price of its good. What if Firm 2 instead chooses a market share of ½? The consumers at address ½, and all the consumers with a higher address, prefer the outside good to good 1. That means that Firm 2 only has to provide this consumer with the zero surplus she would obtain from the outside good, her most preferred alternative. By choosing a market share on which good 1 is considered inferior to the outside good, Firm 2 can price as a monopolist constrained to that interval would, and extract the entire surplus from the marginal consumer.
It is easily verified that when $p_1 = V - tb$, the graphs cross at $b^6$. Of course, when $x=1$, and the market share of Firm 2 is zero, the monopoly and duopoly profit functions are also zero. The slope of the duopoly function at $b$ is less negative than its monopoly counterpart at $b$, and so provided that the kink is to the right of the optimal monopoly market share (on the descending portion of the quadratic), there will be a mirror market share (equivalently, a mirror price) for $b<x<1$. Additionally, the profit at $b$ will be higher than any profit from a higher $x$. Since the optimal monopoly share $x_M^* = 1 - \frac{1}{2} \frac{V}{t}$, we require that $b > 1 - \frac{1}{2} \frac{V}{t}$. Recall that our benchmark duopoly model assumes that $V/t>3/2$. This being the case, at its most binding, this restriction asks that $b>1/4$.

**IV.2. Relevance of the Mirror Price**

If the advertising firm needs to constrain Firm 2, it will be because it wishes its rival to obtain a smaller than optimal market share, that is, $x > x_D^*$. Were this not the case, Firm 2 would automatically accommodate, albeit leaving the market not entirely covered. The implications for the construction of a barrier are clear.

Suppose the support of the advertising campaign is the interval $[a,b]$, $a<b<1$. Given the purpose of the barrier, which is to cordon a segment of the consumer line for Firm 1, Firm 2 has two choices. It may accommodate and accept a market share between 0 and 1-$b$, or it may attempt to price-cut its way past the ads. Let the optimal price for Firm 2 to set while accommodating be called $p_2^*$. Firm 2 will be willing to lower its price to any level up to and including the mirror price, $p_2^*$, since any such successful undercutting will yield a profit weakly greater than the best outcome from accommodation. This

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$^6$ $\pi_M^D = (V - t(1-x))(1-x)$, and $\pi_D^2(p_1, x) = ((2x-1)t + p_1) - x$. When $p_1 = V - tb$ and $x=b$, both expressions are equal to $(V - t(1-b)(1-b))$. 

threat price represents the non-advertiser’s most harmful credible attempt at ignoring the ads.

To prevent this undercutting, Firm 1 must provide consumers on the support of the advertising campaign equal to the surplus they would receive from Firm 2, should it charge the threat price. This gives us the height of the barrier. What of its extent?

Since advertising is costly, Firm 1 will wish to keep the length of the barrier at the minimum level necessary for it to be effective.

The forces governing the shape of the barrier are now known, and we may proceed with more formality to its construction.

IV.III. The Setting

As before, a unit mass of consumers is uniformly distributed along a line of unit length. The advertising firm, Firm 1, is located at address 0. Its rival, Firm 2, is at address 1. Consumers have Hotelling preferences for each good, as given earlier.

\[
U_r^1(r) = V - tr - p_1 \\
U_r^2(r) = V - t(1-r) - p_1
\]

Each consumer buys one of either Firm 1’s good, Firm 2’s good, or an outside good providing a utility of zero.

**Definition:** Monopoly profits at \( r \)

*Monopoly profits for Firm at* \( r \) *are defined as* \( \pi^M_1(r) \equiv (V - tr)r \) *for Firm 1, and* \( \pi^M_2(r) \equiv (V - t(1-r))(1-r) \) *for Firm 2. They are equal to the profits obtained by a non-
advertising monopolist bound to a marginal consumer\textsuperscript{7} at address r, who it leaves indifferent to the outside good.

**Lemma 1**: \( \pi_i^{M}(r) \) is equal to the maximum no-advertising duopoly profit with an indifference point at r.

**Proof**: Given a marginal consumer at r (and assuming the market is covered), the consumer at r is indifferent between the goods provided by firm 1 and firm 2:

\[ U_r^1(r) = U_r^2(r) \]

Expanding the left-hand side, we have

\[ V - tr - p_1 = U_r^2(r) \]

\[ p_1 = V - tr - U_r^2(r) \]

giving us a duopoly profit for firm 1 of

\[ \pi_i^0(r) = p_1r = (V - tr - U_r^2(r))r \]

But, given the existence of the outside good,

\[ \min U_r^2(r) = 0 \]

and so

---

\textsuperscript{7} By marginal consumer, I mean the consumer whose address marks the boundary of a given firm’s market share. The marginal consumer is indifferent between the good of the firm in question and the next-best alternative.
By symmetry, the same is true for Firm 2.

IV.IV. The Barrier

The firms play a two-stage game. In the first stage, Firm 1 sets up the barrier. In the second, the two firms set prices simultaneously.

For the purposes of stage 2, the barrier is an interval [a,b], with a<b<1, which’s consumers are not available to Firm 2. The length of the barrier is defined as \( k \equiv b - a \).

We begin with stage 2, and proceed by backwards induction.

**Proposition 2:** Given \( \frac{1}{3} \frac{V}{t} < b < \frac{1}{2} \frac{V}{t} \),

\[
    p_1 = V - tb \\
    p_2 = V - t(1 - b)
\]

is a Nash equilibrium of the stage 2 game.

**Proof:**

*Part 1: Firm 1’s best response to \( p_2 = V - t(1 - b) \) is to set \( p_1 = V - tb \)

Suppose \( p_2 = V - t(1 - b) \). Then, all consumers on the interval [0,b) prefer the outside good to good 1. It is easily seen that

\[ \max \pi_1^D(r) = (V - tr) r = \pi_1^u(r) \]

The right-hand-side restriction merely says that b is less than that address which yields maximum monopoly profits. While intuitively plausible and supported by the results of numerical simulations, it remains for the moment a conjecture and assumption, rather than a proven result.
\[
\frac{d}{dr} \pi_1^M(r) = V - 2tr > 0 \quad \forall r < \frac{1}{2} \frac{V}{t}
\]

Combining this with Lemma 1, we find that Firm 1 will prefer to set \( p_1 = V - tb \) and earn \( \pi_1^M(b) \) to setting any higher price (and hence lower address of indifferent consumer).

What of lower prices, and indifferent consumers on the interval \((b,1]\)? Given \( p_2 = V - t(1 - b) \), the indifference point between Firms 1 and 2 for \( p_1 < V - tb \) is defined by

\[
V - t(b + \varepsilon) - p_1 = V - t(1 - (b + \varepsilon)) - (V - t(1 - b)) \quad 0 < \varepsilon < 1 - b
\]

This yields

\[
p_1 = V - tb - 2t\varepsilon
\]

and the profits to firm 1 when extending past \( b \) are given by

\[
\pi_1^D(b + \varepsilon) = p_1(b + \varepsilon) = (V - tb - 2t\varepsilon)(b + \varepsilon)
\]

The loss from extending past \( b \) is then given by

\[
L(b, \varepsilon) \equiv \pi_1^{IV}(b) - \pi_1^D(b + \varepsilon) = \varepsilon(2t\varepsilon - V + 3tb)
\]

\[
L(b, \varepsilon) > 0 \quad \forall b > \frac{1}{3} \frac{V}{t}, \varepsilon > 0
\]

and since
\[ \frac{\partial L}{\partial \epsilon} = 3tb + 4t\epsilon - V > 0 \quad \forall b > \frac{V}{3t}, \epsilon > 0 \]

Firm 1 will always prefer charging setting \( p_1 = V - tb \) and earning \( \pi_1^M(b) \) to all other outcomes, given \( p_2 = V - t(1-b) \).

Part 2: Firm 2’s best response to \( p_1 = V - tb \) is \( p_2 = V - t(1-b) \).

When \( p_1 = V - tb \), all consumers to the right of \( b \) prefer the outside good to Firm 1’s good.

By construction (through the definition of the purpose of the barrier), when \( p_1 = V - tb \), firm 2 will prefer to set \( p_2 = V - t(1-b) \) to charging any lower price.

Note that

\[ \pi_2^M(b) < \max \pi_2^M(r), \quad 0 < r < 1 \]
\[ b > \arg \max \pi_2^M(r), \quad 0 < r < 1 \]

If this were not the case, there would be no need for the barrier, as Firm 2 would automatically accommodate and not be tempted to extend into Firm 1’s chosen turf.

It is easily found that

\[ r^* = \arg \max \pi_2^M(r) = 1 - \frac{V}{2t} \]

and
Combined with Lemma 1, we see that firm 1 can earn no profit higher than $\pi_2^M(b)$ by charging a higher price (and obtaining an indifference point to the right of b).

When $p_1 = V - tb$, Firm 2 will not wish to set a price lower or higher than $p_2 = V - t(1 - b)$, q.e.d.

**Stage 1: Construction of the barrier.**

As in previous sections, targeted persuasive advertising allows Firm 1 to pay $c > 1$ to raise preferences for its good by a mass of $1^9$.

The purpose of the barrier is to eliminate the incentive for Firm 2 to extend into Firm 1’s chosen turf, the interval $[0, b)$, given that firm 1 does not itself stray. More precisely, the length of the barrier has to be such that

$$\max_{b \leq r < 1} \pi_2^M(r) \geq \max_{0 < r < b} \pi_2^k(r)$$

where $\pi_2^k(r)$ are the profits obtained by Firm 2 given the existence of a barrier of length k, and that $p_1 = V - tb$.

If $p_1 = V - tb$, the indifference point $x$ between firms 1 and 2 for an arbitrary $p_2$ is defined by

$$V - tx - (V - tb) = V - t(1 - x) - p_2$$

---

9 For instance, to raise preferences for good 1 by 1 on the interval $[0.3, 0.5]$ would cost $cx(0.5 - 0.3)x1 = 0.2c$. 
This gives us $p_2(b, x)$, allowing us to construct

$$\pi_2^k(b, k, x) = p_2(b, x)(1 - x - k)$$

We can solve the first-order conditions for $x^*(b, k)^{10}$, and substitute this back into $\pi_2^k(b, k, x)$ to obtain

$$\max \pi_2^k(b, k) = p_2(b, x^*(b, k))(x - k)$$

Setting this equal to $\pi_2^M(b)$, we can solve for

$$k^*(b) = \min_k \left[ \max \pi_2^k(b, k) = \pi_2^M(b) \right]$$

This is the optimal length for the barrier, insofar as it is the shortest, and therefore cheapest, length of consumers which must be removed from Firm 2’s set of possible customers to entice it to stay on its own turf, when Firm 1 does the same.

The consumers in the blocked interval $[b-k, b]$ must be targeted with persuasive advertising to such an extent that Firm 2 cannot profitably undercut its way past the barrier.

Define

---

10 The attentive reader may wonder whether $0 < r^* < b-k$, as the equations assume. This will be true if $3-3b+2k < V/t < 3-2k+b$. Since generally $k/b$ will be small (that is what makes the barrier effective), this may be loosely interpreted as restricting $V/t$ to be less than 3 and greater than $3-3b$. Note that for the benchmark Hotelling duopoly case to be valid, we require $V/t > 3/2$. Numerical simulation shows that these constraints are seldom binding.
\[ \pi_2^D(b, x) = p_2(b, x)(1 - x) \]

where \( p_2(b, x) \) is as above. This is the profit that Firm 2 can expect after successfully undercutting past the block, for \( p_1 = V - tb \). The equation

\[ \pi_2^D(b, x) = \pi_2^M(b) \]

has two solutions for \( x \), one of which is \( x=b \). The other solution is \( x_T \), the threat point at which Firm 2 charges the lowest price it is willing to offer to overcome the barrier.

\[ x_T = \min_r \left\{ \pi_2^D(b, x) = \pi_2^M(b) \right\} \]

The corresponding price, which I will call the threat price, \( p_T \), is given by

\[ p_T(b) = p_2(b, x_T(b)) \]

Consumers in the barrier interval must be given a surplus, varying with their address \( r \), of

\[ V - t(1 - r) - p_T(b) \]

Prior to advertising, for \( p_1 = V - tb \) they receive

\[ V - tr - (V - tb) \]

The amount to be ‘topped up’ by advertising is the difference between the two:

\[ \tau(b, r) = V - p_T(b) + t(2r - (1 + b)) \]
and the cost of the barrier is the cost of providing this top up over the length of the barrier interval:

\[ \mathcal{A}(b) = \int_{b-k(b)}^{b} \tau(b, r) dr \]

Firm 1’s problem then becomes one of finding the \( b \) to maximize

\[ \pi_1(b) = (V - tb)b - \mathcal{A}(b) \]

This is easily solved numerically. An example follows, with all results given to two significant figures.

**IV.V. Numerical Example**

Suppose \( V=1.8 \), \( t=1 \), and \( c=2.5 \). The optimal \( b \) is then 0.57. This is within the restrictions mentioned above. Firm 2’s threat point is 0.31, which is safely between 0 and \( b-k \), since \( k=0.03 \).

<table>
<thead>
<tr>
<th>Barrier</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>0.67</td>
<td>0.59</td>
</tr>
<tr>
<td>Price</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>Market Share</td>
<td>0.57</td>
<td>0.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Price</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Market Share</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The diagram on the right shows profits for the example as a function of \( b \), in units of benchmark (Hotelling duopoly) profits. The green line is a visual aid for determining
market shares at which advertising is profitable. The red line shows profits for Firm 1, and the blue line for Firm 2.

Compared to the benchmark, both firms benefit, despite Firm 2’s losing market share. The reason that Firm 2 sees its profits increase from the benchmark case is that given the barrier, it is free to monopolize what’s left of the consumer line after Firm 1 has staked out its turf (which it also monopolizes).

The barrier is rather inexpensive – its costs of construction are only 5.6% of Firm 1’s profits, and it extends through 5.3% of Firm 1’s turf.

The figure above shows, given \( t=1 \), the range of \( b \)’s and \( V \)’s for which Firm 1’s profits after the barrier are greater than the benchmark duopoly profits, for \( c=1 \) to \( 8 \). The cost parameter is decreased through five equal intervals in chromatic order, with red being 8 and purple being 1. As costs increase, the viable area shrinks to a subset of its predecessor. The darker areas are those in which Firm 2’s profits also increase. The lighter areas are those in which \( b \) lies between \( V/3t \) and \( V/2t \). Note that the upper constraint never binds. The bright red segment shows the intersection of these last two regions, where \( b \) is within the assumed interval, and Firm 2’s profits increase from the

*Regions compatible with a barrier*
benchmark case. Though this is a small region, Firm 2 will never be entirely driven out of business through advertising, or indeed have less than a 10% market share.

The graphs below give some idea of the magnitudes involved. Again, the height of the graph shows profits in units of Hotelling duopoly profits. Costs have been fixed at 2.5, and \( t=1 \).

---

Firm 1 Profits

Firm 2 Profits
Back to our example. What happens to surplus?

<table>
<thead>
<tr>
<th></th>
<th>Consumers’ Surplus</th>
<th>Producers’ Surplus</th>
<th>Total Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier</td>
<td>0.29</td>
<td>1.2</td>
<td>1.5</td>
</tr>
<tr>
<td>Hotelling duopoly</td>
<td>0.80</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Hotelling monopoly</td>
<td>0.41</td>
<td>0.81</td>
<td>1.2</td>
</tr>
</tbody>
</table>

For our test case, creating a barrier through targeted advertising is even worse for Consumer Surplus than a Hotelling Monopoly – this is not entirely surprising, given the double monopolization. Total surplus is somewhat higher than that in a Hotelling monopoly, but considerably lower than a Hotelling duopoly’s. Numerical simulations suggest these relationships are quite general. Below, Consumer’s, Producer’s and Total surpluses under barrier advertising are plotted for costs of 2.5 when t=1, in units of their benchmark duopoly counterparts. Note that advertising IS considered part of Consumer Surplus – a graph of Consumer Surplus with advertising subtracted from it is included for comparison.
Producer’s Surplus

Total Surplus (Advertising Included)
V. Conclusion

In the preceding sections, we have seen that when given the ability to mold consumer preferences, a monopolist will tend to homogenize the tastes of its least valuable customers. The addition of a rival selling a differentiated product makes this more difficult, but even if homogenization is forbidden (perhaps by law, perhaps by popular action) a firm that can target persuasive advertising will still be able to use this technology to its advantage. It will set up a barrier dividing the consumer line into two neat segments. The advertising firm will monopolize the choice cut, and its rival will gladly monopolize the rest. Its rival may even, under plausible conditions, benefit from the other firm's advertising campaign, relative to the case of an adless duopoly. This outcome will be worse for consumers even than a true monopolist (sans advertising). There is cause, then, to believe that there can be too much information. Privacy advocates are justified in fearing that personal data and purchasing histories gathered by firms may be used to their detriment.

It is interesting that in the duopoly case analyzed, the advertising used may seem to observers to be of an entirely different type. Since the outcome predicts that marginal consumers will be bombarded with advertising while others receive less or none, it may look like a campaign to convince people to switch. Instead, it is better interpreted as a signal of intent, and commitment to stay on one side of the boundary thus marked, and let its rival take the rest. This is seen in the real world in Apple Computer's popular 'switch' campaign. Apple Computer holds 2 to 3% of the market for computer sales (Fried 2003). Its 'Switch' advertising campaign consists of intensive, often-emotional ads targeted towards those people who are (barely) in the PC camp. The campaign consists of vignettes about people who have switched to an Apple computer and are happy because of it. Targeting of the ads can take place not only by choosing where these ads are placed (as banners on web sites devoted to Apple vs. PC comparisons, for instance) but by tailoring the stars of the stories to appeal specifically to the desired demographic. Despite its small market share, Apple computers are rather more expensive than their PC counterparts. All this is consistent with and explained by barrier advertising, which
predicting heavy advertising to the marginal consumers and a monopolization of the captive customer base. The reason that the other 98% of computer buyers aren't similarly gouged is that by comparison with the makers of the Macintosh, PC manufacturers are a competitive fringe.

"Yes, I’m a PC guy that switched to Mac."

— Aaron Adams, Windows LAN administrator

Apple’s Switch campaign: http://www.apple.com/switch/

The model is also useful as an instance of what may happen when a bricks-and-mortar retailer, that is, one without an Internet presence, competes with a rival who only operates online. The online retailer is better-placed to have precise information on its clients, since it can examine and record every aspect of their visits and choices (and beyond, with spyware). The offline store will have more difficulty in retrieving this information, if it may do so at all. Thus, the online store will be able to target its advertising directly, whereas its bricks-and-mortar counterpart will have to make do with no ads, mass advertising, or perhaps crude targeting (ads appealing to large groups such as teens, mothers, the elderly, etc.).

With some modifications, the story also applies to the motivating example referenced in the introduction, of a multinational company entering a new market, and using its accumulated knowledge of consumer behaviour to alter existing preferences in its favour. Homogenization of culture is a danger, though complete homogenization is not, and it is not clear that the local rivals of the multinational, even if (especially if?) they cannot afford a similar advertising technology, will do badly. They may even benefit with comparison to the no-advertising case, where the newcomer 'plays fair'.
Targeted advertising may be used as a tool to grant monopoly power not only to the firm implementing it, but to its rivals. What, then, should be done about it? Privacy regulations may help. The United States already has an opt-in system for telephone marketing, and it could be used as the blueprint for a system to deal with the release and use of all sorts of consumer information by firms. The problem with this is that consumers are likely to opt in to any such program - under barrier advertising, the heavily targeted consumers are very well treated, indeed. Another idea would be to ban direct marketing, but this, too, might fail. Barrier advertising is very robust to 'trembling hand' errors, and does not require precise information to improve the advertising firm's profits over those earned in its absence. (Though of course precise information is needed to reach the optimal point.) By following Apple's lead and advertising heavily to the desired boundary, a 'second-best' outcome may be reached - perhaps making the non-advertising firms worse off in the process.

To properly understand the situation and give advice on how to deal with it from the privacy, regulatory and sociopolitical (cultural diversity) standpoints, the following extensions must be made:

1. The possibility of mass advertising and coarse targeting of consumers (by interval, rather than address) must be introduced into the model. This is necessary both to study the implications of certain regulatory possibilities, and to ascertain how a firm which can advertise, but does not have perfect access to consumer information, might react to a rival so gifted. These will be billboards to targeted advertising's traveling salesman.

2. Both firms must be allowed to advertise. It is likely that this case will yield results similar to the customized pricing literature, in which there is no pure strategy equilibrium and firms vary their promotions. However, it is in the nature of advertising that it takes time to set up, and so the sequence of actions will be important in determining equilibria not seen when coupons are issued.
3. Entry must be examined, both of advertising and non-advertising firms. Can this sort of advertising be used as a barrier to entry, or as an incentive to accommodate upon entry?

The increased availability of information creates both new possibilities and new concerns, and it is important to understand both of these if we are to make the most of what is quickly becoming a truly Known World.

References


